A New Ray-Tracing Program Capable of Simulating Insertion-Device Synchrotron Radiation Sources

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A new ray-tracing program was developed to facilitate optical design of synchrotron radiation beam lines and monochromators. The program uses Monte Carlo modelling to simulate characteristics of insertion-device sources such as undulators and wiggler devices as well as those of normal bending magnets. The program is successfully applied to the optical design of monochromator systems for a new multipole wiggler/undulator beam line (BL16) at the Photon Factory.

KEYWORDS: ray tracing, synchrotron radiation, undulator, multipole wiggler, bending magnet

§1. Introduction

The ray-tracing technique has long required the ability to treat the characteristic properties of synchrotron radiation sources. The most important such property of the ray-tracing standpoint is that the sources have characteristic ray density distributions. This is especially true for highly brilliant insertion-device sources. The increased implementation of insertion-device sources in most synchrotron radiation facilities has prompted the development of a new ray-tracing program capable of treating these source characteristics.

The treatment of source characteristics is well established based on phase space formulations. However, this treatment only applies to normal bending magnet sources. The treatment of multipole wiggler and undulator sources remains unclear. SHADOW is one of the widely available ray-tracing programs for designing synchrotron radiation optics. However, it cannot be used to simulate undulator radiation sources.

This paper reviews the formulations for treating the source characteristics. It describes some examples of source simulations based on Monte Carlo modelling of photon emission along the orbits of an electron beam in the source devices. These examples successfully reproduce the source characteristics of a newly developed multipole wiggler/undulator (BL16 MFW) at the Photon Factory. The ray-tracing program’s structure and utilities are also reviewed.

§2. Simulations of Synchrotron Radiation Sources

2.1 Optical Characteristics of Synchrotron Radiation Sources

The optical characteristics of synchrotron radiation sources depend on the spatial distribution of electron beams in orbit and on the spatial characteristics of the photon emission from the electrons. The optical emittance of synchrotron radiation propagated along the x-axis is defined as a volume in (x, y, y') phase space. Phase space means that the distributions of the radiation beams in x and y positions and in x' and y' angles move along the ideal orbit. In general, the photon emission distributions can be represented by Gaussian forms, the use of which simplifies discussion of the optical properties of synchrotron radiation sources. The distribution probability of a radiation source in a storage ring, f(x, y), can be represented by the double Gaussian,

\[
f(x, y) = \frac{1}{2\pi \Sigma_x \Sigma_y (1 - r^2)^{1/2}} \exp \left[ - \frac{1}{2(1 - r^2)} \right] \times \left\{ \begin{array}{l}
\frac{x^2}{\Sigma_x^2} + \frac{2xy}{\Sigma_x \Sigma_y} + \frac{y^2}{\Sigma_y^2} = 1 - r^2 \end{array} \right\}.
\]

Here \( \Sigma_x \) and \( \Sigma_y \) are effective source sizes for x and y, and r is an asymmetry parameter. The source distributions can then be described in terms of elliptical contours. One of the contours is

\[
\frac{x^2}{\Sigma_x^2} + \frac{2xy}{\Sigma_x \Sigma_y} + \frac{y^2}{\Sigma_y^2} = 1 - r^2.
\]

The distribution probabilities of phase spaces for horizontal x-x' and vertical y-y' spaces are also described by the same double Gaussian forms by using \( \Sigma_x \) and \( \Sigma_y \). These are the effective radiation angles for x and y respectively. Phase space distributions are also described by the same elliptical contours. Therefore the four efficient parameters \( \Sigma_x, \Sigma_y, \Sigma_x, \) and \( \Sigma_y \) accurately represent the spatial behavior of rays radiated from the electron beams.

2.2 Undulator Source

The size of undulator sources depends on electron beam size, source size of photons, and an undulator length. The radiation angle of undulators depends on electron beam divergence and the radiation angle of photons. Therefore, the effective source size \( \Sigma_x, \Sigma_y \) and radiation angles \( \Sigma_x, \Sigma_y \) of undulators can be represented as

\[
\Sigma_x = (\sigma_x^2 + \sigma_y^2 + L^2 \Sigma_x^2 / 4)^{1/2},
\]

\[
\Sigma_y = (\sigma_x^2 + \sigma_y^2 + L^2 \Sigma_y^2 / 4)^{1/2},
\]

\[
\Sigma_x = (\sigma_x^2 + \sigma_y^2)^{1/2}.
\]

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\[ \Sigma_y = \left(\sigma_y^2 + \sigma_y^3 \right)^{1/2}. \]  

Here \( \sigma_x \) and \( \sigma_y \) are the electron beam sizes of \( x \) and \( y \). \( \sigma_x \) and \( \sigma_y \) are the electron beam divergences of \( x \) and \( y \). \( \sigma_x \) and \( \sigma_y \) are the source size and radiation divergence of the photon. \( L \) is the undulator length. \( \sigma_x \) and \( \sigma_y \) are represented as
\[ \sigma_x = \left(\lambda_s L / 2kN \right)^{1/2}, \]
\[ \sigma_y = \left(\lambda_s / L \right)^{1/2}. \]

Here \( \lambda_s \) is the wavelength of the \( k \)-th order harmonic of undulator radiation. \( \gamma \) is the electron energy in rest-mass units. \( N \) is the period number of the magnetic fields in the undulator.

The ray-tracing program can generate rays according to these relations using Monte Carlo modeling. In this model, the rays are generated from random values by the Box and Muller method which is usual in Monte Carlo simulations. Because the \( x-y \) source size, the \( x-x' \) horizontal phase space, and the \( y-y' \) vertical phase space are all represented by double Gaussians, ray positions and radiation angles can only be described by two variables in the double Gaussian distributions generated by the method. One variable is used for \( x \) and \( y \), and another is used for \( x' \) and \( y' \). Therefore, the positions \( (x, y) \) and the radiation angles \( (x', y') \) of the rays can be determined by
\[ x = \Sigma_x (-2 \ln u_1)^{1/2} (1 - r^2)^{1/2} \cos 2\pi u_1 + r \sin 2\pi u_2, \]
\[ y = \Sigma_y (-2 \ln u_1)^{1/2} \sin 2\pi u_2, \]
\[ x' = \Sigma_x (-2 \ln u_1)^{1/2} \sin 2\pi u_2, \]
\[ y' = \Sigma_y (-2 \ln u_1)^{1/2} (1 - r^2)^{1/2} \cos 2\pi u_1 + r \sin 2\pi u_2. \]

Here \( u_1 \) and \( u_2 \) are random values between 0 and 1. The source positions are set in the central plane of the undulator because the undulator radiation is coherent and symmetrical to the optical axis.

Figure 1 shows a spot diagram which represents an actual undulator radiation source image. It also shows an \( x-x' \) horizontal phase space projected onto an image plane in the center of the multipole wiggler/undulator device (BL16 MPW) in the Photon Factory. The device's magnetic field is 0.1 tesla. Its field parameter \( K \) is 1.12. Effective source sizes and effective radiation angles are \( \Sigma_x = 0.824 \) mm, \( \Sigma_y = 0.233 \) mm, \( \Sigma_x = 0.308 \) mrad, and \( \Sigma_y = 0.0374 \) mrad. The images of the undulator source and the phase space are represented as an ellipse symmetrical to the optical axis. This reflects real undulator radiation characteristics, because undulator radiation is coherent and radiation angles are symmetrical to the optical axis.

### 2.3 Wiggler Source

The wiggler source can be regarded as a series of multiple bending magnets, since wiggler radiation is not coherent like undulator radiation. Wiggler radiation source size at an arbitrary position in an insertion device depends on electron beam size and source size of photons. Wiggler radiation angle depends on electron beam divergence, radiation divergence of the photons, and the wiggled electron crossing angle. Therefore, the wiggler radiation's effective source size and radiation angles can be represented as
\[ \Sigma_x = (\sigma_x^2 + \sigma_x^3)^{1/2}, \]
\[ \Sigma_y = (\sigma_y^2 + \sigma_y^3)^{1/2}, \]
\[ \Sigma_x = (\sigma_x^2 + \sigma_x^3 + \psi_0^2)^{1/2}, \]
\[ \Sigma_y = (\sigma_y^2 + \sigma_y^3)^{1/2}. \]

Here, \( \psi_0 \) is the angle at which the wiggled electron crosses the \( z \)-axis. \( \sigma_x \) of the wiggler radiation of photon energy \( E \) is represented as
\[ \sigma_x = 0.565^{-1}(Ec/E)^{0.425}, \]
where \( Ec \) is the critical energy.

The source points are set on the maximum amplitude points of the wiggled electron orbit along the source device in the program. This is done because, as stated,
the wiggler source can be regarded as multiple bending magnets, and the source depth is equal to the device length. Figure 2 shows a spot diagram and an x-y' horizontal phase space projected onto an image plane in the center of the BL16 MPW. The MPW's magnetic field is 1.5 tesla; its field parameter is 16.8; critical energy is 6.19 keV; photon energy is set at 1000 eV. Effective source sizes and effective radiations angles are $\Sigma_x=0.661$ mm, $\Sigma_y=0.241$ mm, $\Sigma_{\varepsilon}=3.45$ mrad, and $\Sigma_{\phi}=0.251$ mrad. The images of the wiggler source and the phase space are represented as a rotating ellipse symmetrical to an electron beam orbit, and agree with the SHADOW\textsuperscript{4} simulation. This behavior reflects definite wiggler radiation characteristics.

2.4 Bending Magnet Source

The effective source size and radiations angles of an electron from bending magnets are basically the same as from wiggler. However, the x' horizontal divergence of the bending magnet is different from that of the wiggler because of curvature of the electron orbit.

The program can generate rays radiated from one electron by the same method as for the wiggler, using Monte Carlo modelling. However, the electron can be randomly set on a curved trajectory, so the x' horizontal divergence is determined by the effective radiation angle and the trajectory angle. The length of the trajectory segment can be arbitrarily determined according to the aperture angle of a beam line. Figure 3 shows a spot diagram and an x-x' horizontal phase space projected onto an image plane in the central position of the radiation source. The magnetic field of the bending magnet is 0.963 tesla; the radius of curvature is 8.66 m; the photon energy is set at 1000 eV. Effective source size and radiation angles are $\Sigma_x=0.665$ mm, $\Sigma_y=0.233$ mm, $\Sigma_{\varepsilon}=0.290$ mrad, and $\Sigma_{\phi}=0.208$ mrad. The trajectory segment ranges from −0.2 m to +0.2 m and the horizontal aperture angle is 46 mrad. The spot diagram shows a horizontally wide source image. The horizontal phase space shows the arch of the phase space. These results are reasonable because the rays radiated from the trajectory segment range were projected onto the image plane at $s=0$. This is characteristic of real synchrotron radiation from a bending magnet.

§3. Program Structure and Utilities

This program was written in FORTRAN77 with a free format. It runs on a FACOM M360P OSIV/F4 MSP system in the Photon Factory computer center. The program interface is simple and easy to use because the program works interactively on a TSS. CPU time is 100 seconds or less. The time from parameter input to data output is about 10 minutes per job.

Fig. 2. A ray-tracing example of wiggler radiation from BL16 MPW. A spot diagram (A) and a horizontal phase space (B) in an image plane in the MPW center.

Fig. 3. A ray-tracing example of synchrotron radiation from a bending magnet. A spot diagram (A) and a horizontal phase space (B) in an image plane in the radiation source center.
This program has 3 parts: the main program, a submain program, and a group of optical element subroutines. The main program calls the submain program, which represents an optical arrangement from a radiation source to an image plane. There are no limits to the optical arrangement. The submain program calls the optical element subroutines according to the arrangement. The optical element subroutines consist of the radiation source, mirror, grating, crystal, Fresnel zone plate, other elements such as the slit and pinhole, and coordinate transformation subroutines. The radiation source subroutines include an undulator, a wiggler, a bending magnet and ordinal radiation sources. 2601 rays are generated in each source subroutine. The mirror subroutines include almost all types of mirrors usually used in synchrotron radiation beam lines and monochromators. The grating subroutines include a constant spacing plane grating and a concave grating. Positions and vectors of rays which are reflected or diffracted by the optical elements can be calculated with double precision rather than mere approximation.

A spot diagram in an image plane (xy-plane), and profiles along x- and y-axes can be output to a laser printer with vignetttings of percent throughput. Vector distributions \((u, v)\) and phase spaces \((x-u, y-v)\) in the image plane can also be output. These utilities can be used to design beam line optics for synchrotron radiation.

§4. Conclusion

The treatment of the source characteristics of insertion devices such as undulators and wigglers can be clarified by formulations to simulate the spatial behavior of rays emitted from the sources. Monte Carlo modelling is used to simulate the ray tracing of synchrotron radiation from either insertion-device sources or normal bending magnets. Simulations accurately reproduce the source characteristics of a newly developed multipole wiggler/undulator and of normal bending magnets at the Photon Factory.

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References